$$\frac{w(\xi)}{\langle w\rangle} = 2\frac{(1-\xi^2)+\frac{2}{3}\frac{\Theta}{\varphi_0}\tau_W(1-\xi^3)}{1+\frac{4}{5}\frac{\Theta}{\varphi_0}\tau_W}.$$

These data permit investigating heat transfer taking account of the new complex $\Theta\tau_W/\varphi_0$.

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SELF-SIMILAR PROBLEMS OF TURBULENT MIXING AT THE INTERFACE OF COMPRESSIBLE GASES*

V. E. Neuvazhaev

UDC 532.517.4

INTRODUCTION

It is well known that the interface of liquids or gases located in a field of gravity breaks down if a heavy substance is located above a light one. An analogous picture arises in the absence of a gravity field, if the light substance accelerates the heavy one. The theory of turbulent mixing and the corresponding self-similar solution for incompressible liquids are constructed in [1].

For some self-similar problems in gasdynamics there arise conditions leading to turbulent mixing. In the present work, solutions are constructed taking account of turbulent mixing. The article discusses the problem of the motion of two originally cold gases, in one of which there is given a rising evolution of energy, varying in accordance with a power or exponential law. In a self-similar solution at an interface, moving with an acceleration, there appears a discontinuity of the density: a shock wave enters the cold gas, leaving behind it a high (at the interface, infinite) density, while a rarefaction wave is propagated into the energy-evolving gas. The interface is obviously unstable, i.e., the light substance accelerates the heavy one. For this problem, a solution is constructed taking account of turbulent mixing.

The article considers the motion of a gas under the action of an applied pressure, rising either stepwise or exponentially. The surface of the gas, to which the pressure is applied, is free. Such a piston can be obtained where, in a vacuum, the pressure is given (for example, of light). A free surface is unstable with respect to small perturbations. In distinction from known self-similar solutions [2, 3], the solution obtained with turbulent mixing

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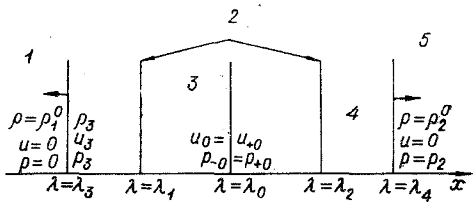

Fig. 1

has an appreciably different distribution of the density and the entropy. The maximal value of the density in a shock wave is attained either at the front or behind the front, but not in the piston, as in a solution without mixing. A minimal, zero, value of the density is attained in a motionless piston.

A shock wave, taking account of mixing, moves with a greater velocity than in the case with mixing. The construction of self-similar solutions with turbulent mixing depends on a constant, determined from experiment. In the examples given below, this constant was borrowed from [1]. Its final choice for a given type of problem can be made by setting up an appropriate experiment.

1. Statement of the Problem with Evolution of Energy. Let the coordinate plane $x = 0$ be the interface between cold quiescent gases with different initial densities and different equations of state:

$$\rho = \rho_1^0;\ p = A_1\rho T,\ \varepsilon = B_1 T \text{ with } x < 0,$$
$$\rho = \rho_2^0;\ p = A_2\rho T,\ \varepsilon = B_2 T \text{ with } x > 0;$$
$$u(o,\ x) = 0,\ T(o,\ x) = 0,$$

where ρ is the density; T is the temperature; p is the pressure; ε is the internal energy; u is the velocity; A_1, A_2, B_1, B_2 are constants; and the subscript 1 relates to the left-hand region and 2, to the right-hand region.

In the region $x > x_0(t)$ [$x_0(t)$ is the trajectory of the interface], in unit mass the evolution of energy is given

$$\varepsilon_0 = \mu F_0 t^n,\ \mu = \begin{cases} 1 & x \geqslant x_0(t) \\ 0 & x < x_0(t); \end{cases}$$

F_0 and n are positive constants.

The equations of gasdynamics in Euler variables have the form

$$\rho(\partial u/\partial t + u\partial u/\partial x) + \partial p/\partial x = 0; \tag{1.1}$$

$$\partial\rho/\partial t + \partial(\rho u)/\partial x = 0; \tag{1.2}$$

$$\partial(\varepsilon + u^2/2)/\partial t + u\partial(\varepsilon + u^2/2)/\partial x + (1/\rho)\partial(pu)/\partial x = \partial\varepsilon_0/\partial t + u\partial\varepsilon_0/\partial x. \tag{1.3}$$

The boundary conditions must be the following: to the left of the energy-evolving region there arises a strong shock wave (Fig. 1, where 1 is the front of the shock wave, 2 is the boundary of the mixing region, 3 is the contact boundary, 4 is the energy-evolving region, and 5 is the rarefaction front):

$$\rho_3 = \frac{\gamma_1 + 1}{\gamma_1 - 1}\rho_1^0,\ p_3 = \frac{2}{\gamma_1 + 1}\rho_1^0 U^2,\ u_3 = \frac{2}{\gamma_1 + 1}U,$$

where $\gamma_1 = 1 + A_1/B_1$; U is the rate of propagation of the shock wave. To the right, along the energy-evolving gas, with the sonic velocity $\sqrt{\gamma_2 p_2/\rho_2^0}$, there moves a rarefaction wave, at whose front

$$u = 0;\ \rho = \rho_2^0,\ p = (\gamma_2 - 1)\rho_2^0\varepsilon_0.$$

At the contact boundary $x = x_0(t)$, the conditions of continuity of the pressure and the velocity are fulfilled:

$$p_{-0} = p_{+0},\ u_{-0} = u_{+0}.$$

The problem posed is self-similar. If we introduce dimensionless variables according to the formulas

$$\lambda = [(n+2)/2](B_2/A_2F_0)^{1/2}x/t^{1+n/2},\ u = (A_2F_0/B_2)^{1/2}t^{n/2}\zeta(\lambda),$$
$$\rho = \rho_1^0\delta(\lambda),\ T = (F_0/B_2)t^n\theta(\lambda),\ p = A_2(F_0/B_2)t^n\pi(\lambda), \tag{1.4}$$

then Eqs. (1.1)-(1.3), after the substitution of (1.4) into them, reduce to a system of ordinary differential equations:

$$N = n/(n+2),\ \delta[N\zeta + (\zeta-\lambda)\zeta'] + \pi' = 0; \tag{1.5}$$

$$(\lambda-\zeta)\delta' = \delta\zeta'; \tag{1.6}$$

$$2N\mu/(\gamma-1) = 2N(\zeta^2/2 + \theta/(\gamma-1)) + (\zeta-\lambda)(\zeta^2/2 + \theta/(\gamma-1))' + \theta\zeta. \tag{1.7}$$

In the last equation, the adiabatic index γ takes on its own value in each region: $\mu = 0$, $\gamma = \gamma_1$; with $\mu = 1$, $\gamma = \gamma_2$.

Correspondingly, the boundary conditions in self-similar variables have the following form:

at the front of the shock wave ($\lambda = \lambda_3$),

$$\delta_3 = \frac{\gamma_1+1}{\gamma_1-1},\ \pi_3 = [2/(\gamma_1+1)]\lambda_3^2,\ \zeta_3 = [2/(\gamma_1+1)]\lambda_3; \tag{1.8}$$

at the front of the rarefaction wave ($\lambda = \lambda_4 = \sqrt{\gamma_2}$),

$$\delta_2 = \rho_2^0/\rho_1^0;\ \zeta_2 = 0;\ \pi_2 = \delta_2; \tag{1.9}$$

and at the contact boundary ($\lambda = \lambda_0$),

$$\zeta_{-0} = \zeta_{+0} = \lambda_0;\ \pi_{-0} = \pi_{+0}. \tag{1.10}$$

If we assume that the energy evolution depends exponentially on the time:

$$\varepsilon = \mu F_0 e^{\beta t},$$

then, introducing the corresponding dimensionless variables

$$\lambda = \left(\frac{B_2}{A_2F_0}\right)^{1/2} xe^{-(\beta/2)t},\ u = \left(\frac{A_2F_0}{B_2}\right)^{1/2} e^{(\beta/2)t}\zeta(\lambda),$$
$$\rho = \rho_1^0\delta(\lambda),\ T = \frac{F_0}{B_2}e^{\beta t}\theta(\lambda),\ p = \frac{A_2F_0}{B_2}\rho_1^0 e^{\beta t}\pi(\lambda),$$

we obtain a system of ordinary differential equations, coinciding with (1.3) if, in these equations, we set N = 1. The boundary conditions are retained without change.

2. Structure of Self-similar Solution. Instability of Contact Boundary. The system of ordinary differential equations (1.5)-(1.7) can be integrated numerically. The known front of the rarefaction wave $\lambda = \lambda_4$ (1.9) for the system under consideration is a singular point. This singularity was investigated in [4]. Through the point there passes a single-parameter family of integral curves. The integration can be carried out by adjustment: start from the rarefaction front $\lambda = \lambda_4$ (using an expansion with some value of the constant c_0), carry it to the contact boundary $\zeta_{+0} = \lambda_0$, and then from the front of the shock wave λ_3 to the contact boundary. The value of λ_3 is so selected that, with $\lambda = \lambda_0$, $\zeta_{-0} = \zeta_{+0}$. Here $\pi_{-0} \neq \pi_{+0}$. Continuity of the dimensionless pressure is assured by the selection of the constant in the expansion c_0.

The solution constructed in this manner will consist of two regions: a shock wave (heavy gas) and a rarefaction wave (light gas), separated by the contact boundary. The latter plays the role of a piston. It is known from [2] that the density to the left of the boundary takes on an infinite value. Under these circumstances, the temperature reverts to zero. Since the index of self-similarity is positive ($n > 0$), the contact boundary moves in an accelerated manner. There arises the case where the light gas accelerates the heavy gas. The situation is analogous to Rayleigh–Taylor instability.

3. Turbulent Mixing. Theory and Equations. Instability of the contact boundary inevitably leads to turbulent mixing: a complex motion is formed, with one of the gases penetrating into the other. In some cases, a quantitative description of such motions can be obtained by application of known semiempirical theories.

The theory of turbulent mixing was constructed in [1] under the assumption of isothermicity. Its further development and generalization for the case of adiabatic motions are contained in [5].

The coefficient of turbulent diffusion and thermal conductivity is introduced:*

$$D = lv, \tag{3.1}$$

where l is the characteristic turbulent length, connected with the width of the mixing region by the empirical constant α

$$l = \alpha L; \tag{3.2}$$

v is some characteristic turbulent velocity, expressed approximately in terms of the profile of the sought solution by the formula from [5]:

$$v = l\omega, \quad \omega = \sqrt{g\left(\partial \ln \rho/\partial x + g/a_0^2\right)}, \tag{3.3}$$

where g is the acceleration; a_0 is the velocity of sound; $v = 0$, if the expression under the square-root sign is negative; in this case the motion is stable. With a more exact discussion, for the turbulent velocity v the balance equation can be written

$$(1/2)\partial\rho v^2/\partial t + \mathbf{v}\rho v^3/l = \rho l v \omega^2. \tag{3.4}$$

Below a study is made of the simpler case where the time derivative in the left-hand part of (3.4) is neglected. Then instead of (3.4), we obtain (3.3).

We denote the concentration of the active component in the mixture:

$$c = \rho_2/(\rho_1 + \rho_2)$$

($\rho_1 + \rho_2$ is the density of the mixture); we write an equation describing the change in the concentration;

$$\partial c/\partial t + u\partial c/\partial x = (1/\rho)\partial(\rho D \partial c/\partial x)/\partial x. \tag{3.5}$$

In the region of the mixture, as in [5], the equations of state are determined using the formulas

$$p = [A_1(1 - c) + A_2 c]\rho T; \ \varepsilon = [B_1(1 - c) + B_2 c]T, \tag{3.6}$$

where ρ and T are the density and temperature of the mixture.

Finally, in the energy equation (1.3), it is necessary to add the turbulent thermal conductivity, i.e., the main dissipative mechanism leading to diffusion of the entropy and the density. In addition, the fact of mixing must be taken into consideration in the energy-evolution term. Taking account of the above-listed factors, we have,

*As in Russian original — Publisher.

$$\frac{\partial}{\partial t}\left(\varepsilon+\frac{u^2}{2}\right)+u\frac{\partial}{\partial x}\left(\varepsilon+\frac{u^2}{2}\right)+\frac{1}{\rho}\frac{\partial}{\partial x}pu=\frac{1}{\rho}\frac{\partial}{\partial x}\rho D\left(\frac{\partial\varepsilon}{\partial x}+p\frac{\partial\frac{1}{\rho}}{\partial x}\right)+\frac{\partial\varepsilon_0 c}{\partial t}+u\frac{\partial\varepsilon_0 c}{\partial x}. \tag{3.7}$$

To Eqs. (3.5)-(3.7) we add without change the equation of the conservation of motion (1.1) and the equation of continuity (1.2). Thus, we obtain a system of equations describing the motion of a gas caught up in turbulent mixing.

4. Self-similar Character of Problem with Mixing. Examples. The new equations introduced (3.5), (3.7) retain the self-similarity discussed in Sec. 2. In actuality, the width of the mixing zone can be defined, for example, as the distance between the corresponding λ terms (see Fig. 1, λ_1 and λ_2) at which the concentration $c(\lambda_1) = 0$ and $c(\lambda_2) = 1$. Then,

$$L = x_2 - x_1 = [2/(n+2)](A_2F_0/B_2)^{1/2}t^{(2+n)/2}(\lambda_2-\lambda_1); \tag{4.1}$$

λ_1 and λ_2 are as yet unknown. Substituting (1.4) into (3.5), (3.7), and taking account of (3.1)-(3.3) and the expression for the width L (4.1), we finally obtain two ordinary differential equations

$$\begin{gathered}
2Nc/(\gamma_2-1) = 2N(\bar\theta+\zeta^2/2)+(\zeta-\lambda)(\zeta^2/2+\bar\theta)'+(1/\delta)(\bar\pi\zeta-q)',\\
q=\delta d\left[\bar\theta'-\frac{\bar\pi}{\delta}(\ln\delta)'-c'\right];\quad \bar\pi=\left[\frac{A_1}{A_2}(1-c)+c\right]\pi;\\
\bar\gamma=\frac{(A_1+B_1)(1-c)+(A_2-B_2)c}{B_1(1+c)+B_2c};\quad \bar\theta=\left[\frac{B_1}{A_2}(1-c)+\frac{B_2}{A_2}c\right]\theta;\\
d=\alpha^2(\lambda_2-\lambda_1)^2\sqrt{-\frac{\bar\pi'}{\delta}\left[(\ln\delta)'-\frac{1}{\bar\gamma}(\ln\bar\pi)'\right]},\\
\delta c'(\zeta-\lambda)=(\delta dc)'.
\end{gathered} \tag{4.2}$$

These equations hold only in the mixing zone from λ_1 to λ_2, where the coefficient d differs from zero. They go over into the original equations in regions where d = 0.

Figures 2 and 3 give plotted self-similar profiles for $\gamma_1 = \gamma_2 = 7/5$, N = 0.5 and $\gamma_1 = \gamma_2 = 5/3$, N = 1. The initial density in both regions is assumed identical. It was found difficult to carry out numerical integration of the system of ordinary differential equations obtained. The difficulties arising with integration can be followed in the partial case analyzed in Sec. 5. Therefore, the problem was solved in partial derivatives by the method of [5]. Here the difference grid with respect to the spatial variable was selected in such a way that values obtained from an increase in the number of points would vary only slightly (so that they would coincide graphically). For purposes of comparison, the same figures, by the dashed lines, show the solution without mixing. It can be seen from the curves that mixing considerably changed the density of the gas and led to a certain determinacy in the position of the front of the shock wave.

5. The Problem of a Piston. Approximate Solution without Mixing. The problem of a piston with a given boundary pressure

$$p = p_0t^n \tag{5.1}$$

can be regarded as a partial case of the problem of Sec. 1, where the density of the energy-evolving layer $\rho_2^{\circ} \to 0$; under these circumstances, the temperature of the energy-evolving layer should approach ∞ in such a way that relationship (5.1) will hold. The solution of this self-similar problem without mixing is known and was constructed in [2, 3]. Its structure will coincide with the structure of the solution of the starting problem in the part where there is no energy evolution. The boundary with the vacuum, moving with acceleration toward the side of the substance, will obviously be unstable. Therefore, turbulent mixing must be taken into consideration.

Turbulent mixing does not arise if the piston is realized using a solid wall. Numerical integration of the ordinary differential equations obtained can be simplified if, in the energy equation, the derivative of the entropy function is replaced by

$$\partial\ln(p/\rho^\gamma)/\partial r \approx \partial\ln(1/\rho_\gamma)/\partial r.$$

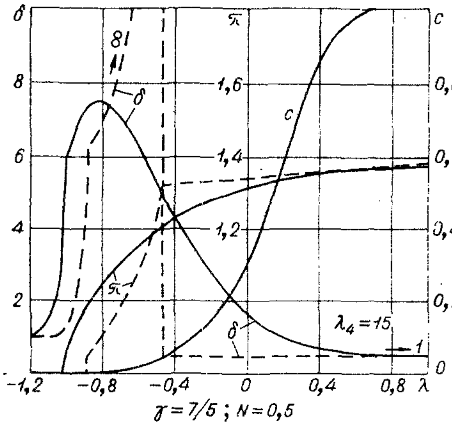

Fig. 2

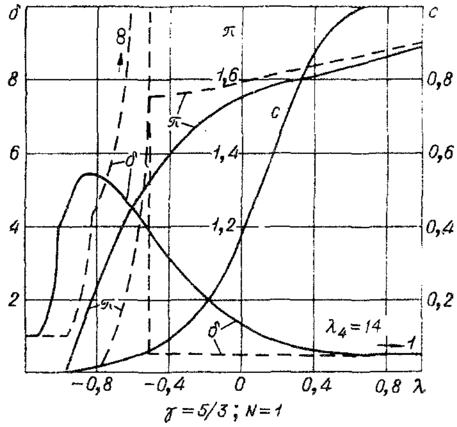

Fig. 3

This can be done on the basis of the fact that the change in the pressure takes place slowly compared with that of the density.

The solution of the problem of a piston without mixing, in the proposed approximation, is obtained in analytical form, with the retention of the special characteristics of the exact solution (an infinite density and a zero temperature at the piston). The problem of integrating the equations in the case of turbulent mixing reduces to the numerical solution of one ordinary differential equation.

The dimensionless variables are connected with the starting variables by the following relationships:

$$\lambda = \frac{n+2}{2}\left(\frac{\rho_1^0}{p_0}\right)^{1/2}\frac{x}{t^{(n+2)/2}}, \quad u = \left(\frac{p_0}{\rho_1^0}\right)^{1/2}\zeta(\lambda),$$

$$\rho = \rho_1^0\delta(\lambda), \ p = p_0 t^n \pi(\lambda), \ T = \frac{p_0}{\rho_1^0 A_1} t^n \theta(\lambda).$$

The first two equations coincide with (1.5), (1.6). In Eq. (1.7), we must set $\mu = 0$ and, after transformations using (1.5), it can be represented in the form

$$a'(\zeta - \lambda) + 2N/(\gamma - 1) = 0, \tag{5.2}$$

where $a = [1/(\gamma - 1)]\ln(\pi/\delta^\gamma)$. Further, we assume

$$a' = [1/(\gamma - 1)][(\ln\pi)' - \gamma(\ln\delta)'] \approx [-\gamma/(\gamma - 1)](\ln\delta)'. \tag{5.3}$$

Substituting (5.3) into (5.2), using (1.6) we obtain an equation for the dimensionless velocity ζ:

$$\zeta' = 2N/\gamma. \tag{5.4}$$

Since the pressure at the piston is known, then boundary condition (1.10) assumes the form

$$\lambda_0 = \zeta_0; \ \pi_0 = 1. \tag{5.5}$$

Equations (5.4), (1.5), and (1.6) can be integrated and satisfy the boundary conditions (1.8), (5.5). The solution is represented in analytical form:

$$K_1 = \frac{2N(\gamma+1) + 2\gamma}{(\gamma+1)(\gamma+2N)}; \quad K_2 = \frac{\gamma + N(2-\gamma)}{\gamma(N+\gamma)};$$

$$\lambda_3 = \sqrt{\frac{\pi_0}{\frac{2}{\gamma+1} + N[K_1 + K_2(1 - K_1)]}},$$

$$\lambda_0 = K_1\lambda_3, \quad \delta = \frac{\gamma+1}{\gamma-1}\left(\frac{\lambda-\lambda_0}{\lambda_3-\lambda_0}\right)^{-2N/(2N+\gamma)}, \tag{5.6}$$

$$\zeta = \lambda_0 - \frac{2N}{\gamma}(\lambda-\lambda_0),$$

$$\pi = \frac{2}{\gamma+1}\lambda_3^2 + N\lambda_3[\lambda_0 + K_2(\lambda_3-\lambda_0)] - N\left(\frac{\lambda-\lambda_0}{\lambda_3-\lambda_0}\right)^{\gamma/(2N+\gamma)} [K_2(\lambda-\lambda_0)+\lambda_0]\lambda_3.$$

To evaluate the legitimacy of approximation (5.3), we calculate the ratio $(\ln \pi)'/(\gamma \ln \delta)'$, using solution (5.6). In the interval $[\lambda_0, \lambda_3]$, the following inequality holds:

$$0 \leqslant (\ln\pi)'/\gamma(\ln\delta)' \leqslant (2\gamma-1)/2\gamma, \tag{5.7}$$

i.e., the assumption of the smallness of the numerator in (5.7) in comparison with the denominator is always true, the more so the smaller the adiabatic index γ.

In the approximate solution, the density at the piston, as in the exact solution, reverts to infinity. The solution obtained (5.6), on the basis of (5.7), can be used also for $N < 0$. Then it is of interest to compare it with L. I. Sedov's exact solution of the problem of a plane explosion. To this end, we must set $N = -0.5$. Here for $\gamma = 1.4$ we obtain $\pi_0 = 0.383$; in the exact solution from [6], we have $\pi_0 = 0.325$. A solution for cylindrical and spherical pistons, in the above approximation, can be obtained in analytical form.

6. Solution of the Problem of a Piston with Mixing. The energy equation (4.2) has the form

$$\pi[a'(\zeta-\lambda) + 2N/(\gamma-1)] = q'; \tag{6.1}$$

$$q = \alpha^2(\lambda_1-\lambda_0)^2\left(-\frac{\gamma-1}{\gamma}\frac{\pi'}{\delta}\right)^{1/2}\pi(-a')^{3/2}.$$

We make two assumptions. In the first place, we apply the approximate equality (5.3); in the second place, in the right-hand part of Eq. (6.1), we take the pressure π out from under the differential sign. We obtain

$$\frac{2N}{\gamma} - (\zeta-\lambda)\delta'/\delta = -\alpha^2(\lambda_1-\lambda_0)^2\left[\left(-\frac{\pi'}{\delta}\right)^{1/2}\left(\frac{\delta'}{\delta}\right)^{3/2}\right]'.$$

This last equation can be integrated if the left-hand part is transformed using Eq. (1.6):

$$\frac{2N}{\gamma}(1-\lambda) + \frac{2}{\gamma+1} - \zeta = \alpha^2(\lambda_1-\lambda_0)^2\left(-\frac{\pi'}{\delta}\right)^{1/2}\left(\frac{\delta'}{\delta}\right)^{3/2}. \tag{6.2}$$

The integration constant is determined from the condition at the boundary of the mixing region:

$$\lambda = \lambda_1, \; q = 0. \tag{6.3}$$

Using (1.5), (1.6), Eq. (6.2) can be reduced to an equation of the first order for the single function ζ:

$$(2N/\gamma)(1-\lambda) + 2/(\gamma+1) - \zeta = \alpha^2(\lambda_1-\lambda_0)^2[N\zeta + (\zeta-\lambda)\zeta']^{1/2}(\zeta'/(\lambda-\zeta))^{3/2}. \tag{6.4}$$

This equation must be integrated in the mixing region $\lambda_0 \leqslant \lambda \leqslant \lambda_1$.

From (6.3), there follows one boundary condition:

$$\lambda = \lambda_1, \; \zeta' = 0. \tag{6.5}$$

The other boundary condition is the piston itself:

$$\lambda_0 = \zeta_0.$$

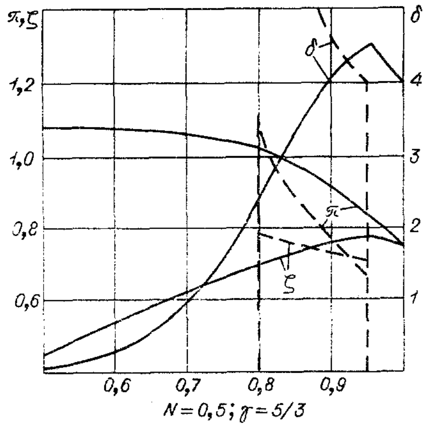

Fig. 4

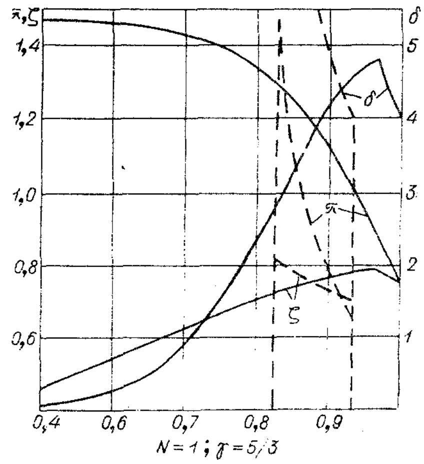

Fig. 5

To this condition there must be added the natural condition of a vacuum:

$$\delta = 0 \text{ with } \lambda_0 = \zeta_0.$$

An analysis of the behavior of the integral curves of Eq. (6.2) leads to a situation in which $\lambda_0 = \zeta_0 \neq 0$ $\delta(\lambda_0) \neq 0$.

Setting $\lambda_0 = 0$, we obtain a single integral curve with the expansion

$$\zeta = (1-N)\lambda + \frac{\left(\frac{2N}{\gamma} + \frac{2}{\gamma+1}\right)^2 N^3}{(1-3N)(1-N)^3\alpha^4\lambda_1^4}\lambda^3 + \ldots,$$

which is the sought solution. In actuality, in this case, the expansion for the dimensionless density δ is

$$\delta = c_0\lambda^{(1-N)/N}.$$

Consequently, $\delta(0) = 0$.

The integration must be carried out up to some point $\lambda = \lambda_1$ at which the condition (6.5) is satisfied. Since the quantity λ_1 enters into the coefficients of the equation, its final determination can be carried out by iterations.

After finding the dimensionless velocity ζ, the remaining functions are calculated using the formulas

$$\delta = \frac{\gamma+1}{\gamma-1}\delta_1 e^{\int_{\lambda^1}^{\lambda}[\zeta'/(\lambda-\zeta)]d\lambda};$$

$$\pi = \pi_1 - \int_{\lambda^1}^{\lambda}\delta[N\zeta + (\zeta-\lambda)\zeta']\,d\lambda,$$

where δ_1, π_1 are the values of the functions with $\lambda = \lambda_1$.

The solution between the front of the shock wave $\lambda_3 = 1$ and the front of the mixing λ_1 is described by the formulas (5.6).

Figures 4 and 5 illustrate the approximate solution of the problem for a piston, taking account of turbulent mixing. For purposes of comparison, the same figures show the solution without mixing. The case N = 0.5 and 1, $\gamma = 5/3$ is considered. The constant $\alpha = 0.133$ was taken from [1]. As for the problem of Sec. 4, the effect of mixing leads to another distribution of the density and to determinacy in the position of the front of the shock wave.

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STEADY-STATE PERTURBATIONS IN A LIQUID CONTAINING GAS BUBBLES

V. V. Goncharov, K. A. Naugol'nykh, and S. A. Rybak

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The problem of wave propagation in a liquid with gas bubbles, which is an example of a nonlinear dispersive medium, is usually treated in the approximation of weak nonlinearity and dispersion [1, 2], and a solution has been successfully obtained only in some special cases corresponding to a strong variation of the bubble radius [3]. In contrast to this, it is shown in this paper that a wider class of solutions is successfully found for stationary waves which correspond to highly nonlinear pulsations of the bubbles. At the same time, periodic solutions appear along with solutions of the soliton type, which correspond to the situation in which nonlinear and dispersive effects just compensate each other.

One-dimensional acoustic waves in a bubble medium can be described by a system of linear acoustic equations which take account of the presence of gas bubbles:

$$\partial\rho/\partial t+\rho_0\partial v/\partial x=0;\ \partial v/\partial t+(1/\rho_0)\partial p/\partial x=0;\ \rho/\rho_0=\left[(1-z)/\rho_0 c_0^2\right]p-nV \tag{1}$$

and by the Rayleigh nonlinear equation for oscillations of a gas bubble

$$Rd^2R/dt^2+(3/2)(dR/dt)^2=(p_0/\rho_0)[(R_0/R)^{3\gamma}-1]-p/\rho_0. \tag{2}$$

Since $p = p(t, x)$, then $R = R(t, x)$ and $dR/dt \approx \partial R/\partial t$ on the condition that one can neglect the convective nonlinear terms. Here ρ_0, p_0, and c_0 are the equilibrium values of the density, pressure, and speed of sound, respectively, in a liquid without bubbles; R_0 is the equilibrium bubble radius; R is its instantaneous radius; γ is the adiabatic exponent for the gas in the bubble; n is the number of bubbles per unit volume; z is the bubble concentration; and ρ, p, v, and V are the variations in the density, pressure, speed of the liquid's particles, and the bubble volume, respectively.

If one introduces the equilibrium bubble volume $V_0 = (4/3)\pi R_0^3$ $[z = nV_0, V_0 + V = (4/3)\pi R^3]$ the eigenfrequency of the oscillations of the bubble $\omega_0^2 = 3\gamma p_0/\rho_0 R_0^2$, and also the dimen-

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